

# Neutrino oscillations as a “which-path” experiment

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## Abstract

The role of simple quantum mechanics in understanding neutrino oscillation experiments is pointed out by comparison with two-slit and Bragg scattering experiments. The importance of considering the beam and the detector as a correlated quantum system is emphasized. Quantum mechanics alone shows that the difference observed in the same neutrino detector at Super-Kamiokande between upward and downward going neutrinos requires the existence of a neutrino mass difference. The localization of the source and detector in space in the laboratory system for long times leads to an uncertainty in the momentum but not of the energy of the neutrino and to coherence between states having different momenta and the same energy and not between states with different energies.

## I. THE BASIC QUANTUM MECHANICS OF NEUTRINO OSCILLATIONS

### A. Coherence and the momentum uncertainty

Coherence and interference in neutrino oscillations have been extensively discussed and clarified [1–11] but there is still considerable confusion. The standard textbook neutrino

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\*Supported in part by grant from US-Israel Bi-National Science Foundation and by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38.

wave function, a coherent linear combination of states with different energies, never exists in the real world. Elementary quantum mechanics and quantum statistical mechanics tell us that components with different energies in an initial state are never coherent [9] while components with different momenta must be coherent. The probability must vanish for finding a neutrino source outside the tiny region of space where the source is known to exist. Any wave packet or density matrix describing the source as a linear combination of plane wave momentum eigenstates which exist over all space with constant amplitudes must somehow conspire to produce this cancellation outside the source. This coherence between states having the same energy and different momenta produces coherence between neutrino states with the same energy and different masses.

## B. Simple Quantum Mechanics and Super-Kamiokande

Simple quantum mechanics alone, without the full apparatus of the standad model, shows that the Super-Kamiokande results [12] require the existence of two different mass eigenstates for neutrinos. The energy spectrum of atmospheric neutrinos cannot change between their source at the top of the atmosphere and their detection in a detector on earth if neutrinos are not absorbed and do not decay en route and any interactions en route conserve energy. The momentum spectrum for neutrinos of a given energy is a set of delta functions, one for each neutrino mass value. If there is only one mass value, the energy and momentum spectra will be identical for the upward and downward going neutrinos incident on the detector and no difference between them can be observed. The observation of such a difference [12] therefore indicates that there are at least two different mass eigenstates, and that the difference can arise from interference between the waves of states having different masses and therefore different momenta if they have the same energy.

### C. The Static Point Source Approximation

If a neutrino is emitted from a point source which is at a definite position in the laboratory for all time, the neutrino energy can be determined precisely by measuring the energy of the source before and after the neutrino emission. But the point source localization introduces an infinite momentum uncertainty. In a realistic case, the source still remains undisturbed for a sufficiently long time on the relevant time scale, and its finite size is still very much smaller than the wave lengths in space of any neutrino oscillation and the distance between the source and the detector. Thus the static point source provides a very good approximation for determining which amplitudes are coherent and which are incoherent. Amplitudes describing neutrino states with the same energy and different momenta are coherent and must be summed before squaring, while amplitudes having different energy are incoherent and are squared before summation. This is discussed quantitatively below.

## II. THE ANALOG WITH TWO-SLIT AND BRAGG SCATTERING EXPERIMENTS

The wave-particle duality and quantum mechanics inherent in a neutrino-oscillation experiment can be clarified by considering it as a typical “which-path” experiment [13]. Just as in the two-slit electron diffraction experiment and in coherent Bragg scattering of photons by a crystal, the neutrino oscillation experiment describes the emission of a particle from a source and its detection by a detector separated from the source by a macroscopic distance. There there is no measurement of the precise path taken by the particle from the source to the detector. The amplitude at the detector is the coherent sum of the amplitudes from all allowed paths.

In the Bragg scattering experiment, the photon may be scattered by any one of the atoms in the crystal, transferring momentum and energy, but which atom scattered the photon is not known. In a neutrino oscillation experiment, the neutrino carrying momentum and energy

from the source to detector may be any one of the allowed neutrino mass eigenstates, but which mass eigenstate carries this momentum and energy is not known. Here the relevant paths are in energy-momentum space, rather than configuration space. It is not simple ignorance which conceals the information on the neutrino mass. Simple ignorance of which path is taken by a particle does not introduce coherence between amplitudes.

Coherence results only from an uncertainty required by quantum mechanics. Both in Bragg scattering and neutrino oscillations there would be no coherence if the energy and momenta of all relevant particles were measured precisely. The positions both of the atoms in the crystal and of the neutrino source in the laboratory are known to a precision which produces a sufficiently large momentum uncertainty to prevent the identification of the scattering atom or of the neutrino mass. These uncertainties prevent the use of momentum conservation to distinguish between different possible amplitudes leading to the same final state at the detector. Because the experimental setup is crucial to the determination of which amplitudes are coherent, the relevant conditions determined by the experimental setup must be introduced into any calculation from the beginning. It is thus desirable to work at all times in the laboratory system, where the source, detector and scattering apparatus are not moving and the constraints from the uncertainty principle are most simply described.

### **III. WHICH PATH OR WITCH CRAFT?**

Further insight into the physics of Which-Path experiments is given by noting the existence of quantum detectors and including the quantum mechanics of the detector in the analysis of the experiment.

#### **A. Classical and Quantum Detectors**

A classical detector in one path of a two-path experiment determines which path was taken and destroys all coherence and interference. A quantum detector is a quantum system in one path of a which-path experiment. If a particle passes through its path, it undergoes

a transition denoted by  $|D_i\rangle \rightarrow |D_f\rangle$ , where  $|D_i\rangle$  and  $|D_f\rangle$  denote the initial and final states of the detector.

Consider a simple “two-slit” which-path experiment in which a quantum detector is introduced [13,14] into one path. A particle beam is split into two paths and the two amplitudes, denoted by  $|L(x)\rangle$  and  $|R(x)\rangle$  are then recombined at a point  $x$  on a screen.

If no path detector is present the wave function and the intensity at the point  $x$  are

$$\Psi(x) = |L(x)\rangle + |R(x)\rangle \quad (3.1)$$

$$I(x) = |\Psi(x)|^2 = ||L(x)\rangle|^2 + ||R(x)\rangle|^2 + 2\text{Re}[\langle L(x) | R(x)\rangle] \quad (3.2)$$

This can be rewritten

$$I(x) = ||L(x)\rangle|^2 + ||R(x)\rangle|^2 + 2\text{Re}[|\langle L(x) | R(x)\rangle| \cdot e^{i\theta(x)}] \quad (3.3)$$

$$I(x) = ||L(x)\rangle|^2 + ||R(x)\rangle|^2 + 2|\langle L(x) | R(x)\rangle| \cos \theta(x) \quad (3.4)$$

where  $\theta(x)$  is relative phase of  $|L(x)\rangle$  and  $|R(x)\rangle$

If there is a quantum detector in the “R” path, the wave function for the combined system of the particle and the detector and the intensity observed at  $x$  are

$$\Psi(x, D) = |L(x), D_i\rangle + |R(x), D_f\rangle \quad (3.5)$$

$$I(x) = ||L(x)\rangle|^2 + ||R(x)\rangle|^2 + 2\text{Re}[\langle L(x) | R(x)\rangle \cdot \langle D_i | D_f\rangle] \quad (3.6)$$

The quantum detector introduces an additional factor in the interference term, the detector overlap  $\langle D_i | D_f\rangle$ . It can also have an additional phase introduced by the phase of  $\langle D_i | D_f\rangle$

The Bragg scattering process is an example of a which-path experiment with many paths, one for each scattering atom, and a quantum detector in each path. The detector is the full lattice and each interference term between two paths contains two coherence factors

$\langle D_i | D_f \rangle$ , one for each path, that depend on the lattice dynamics. The probability  $P_{DW}$  that the scattering is coherent is called the “Debye-Waller” factor [15,14] and is just given by

$$P_{DW} = |\langle D_i | D_f \rangle|^2 \quad (3.7)$$

## B. A Simple Toy Model for a Quantum Detector

Consider the following modification of the toy model of Stern et al [13] in which the particle moving in the “R” path interacts with an external spin- one-half object and produces a rotation of this external spin by exactly  $180^\circ$  about the  $z$ -axis, while if the particle passes through the “L” path there is no effect. Then

$$|D_f\rangle = e^{i\pi s_z} |D_i\rangle = e^{i\pi\sigma_z/2} |D_i\rangle \quad (3.8)$$

$$\langle D_f | D_i \rangle = \langle D_i | e^{i\pi\sigma_z/2} | D_i \rangle = \langle D_i | i\sigma_z | D_i \rangle = i\langle \sigma_z \rangle_i \quad (3.9)$$

The wave function and intensity at the point  $x$  on the screen are now

$$\Psi(x, D) = [|L(x), D_i\rangle + i\langle \sigma_z \rangle_i |R(x), D_i\rangle] \quad (3.10)$$

$$I(x) = ||L(x)\rangle|^2 + ||R(x)\rangle|^2 - 2|\langle L(x) | R(x)\rangle| \sin \theta(x) \cdot \langle \sigma_z \rangle_i \quad (3.11)$$

The interference term with quantum detector contains an additional factor  $\langle D_i | \sigma_z | D_i \rangle = \langle \sigma_z \rangle_i$  with an extra  $90^\circ$  phase.

If the spin is initially polarized in the any direction normal to the  $z$  axis, then  $\langle \sigma_z \rangle_i = 0$  and there is no interference between the two paths. One path flips the external spin; the other does not, and the detector determines the path.

If the spin is initially polarized in the  $z$ -direction, then  $\langle \sigma_z \rangle_i = \pm 1$  and the rotation does not change the spin state; it only introduces a phase. There is no dephasing, just the addition of a constant relative phase.

The interesting case which illustrates the difference between classical and quantum detectors is when the spin is initially polarized in another direction; e.g. at  $45^\circ$  relative to the  $z$ -axis in the  $x - z$  plane, with the  $z$  and  $x$  components both positive. Here  $\langle \sigma_z \rangle_i = 1/2$ .

Classically it is always possible to know the path taken by the particle. If the spin is rotated the “R” path has been taken; if the spin is not rotated, the “L” path has been taken. The rotation brings the spin into a direction in the  $x - z$  plane which is still at  $45^\circ$  relative to the  $z$  axis and normal to the original direction. The  $z$ -component is still positive but the  $x$  component is negative.

This rotation is easily detected classically but not quantum-mechanically. The initial spin state which is 100% polarized positive relative to an axis at  $45^\circ$  with respect to the  $z$  axis with both  $x$  and  $z$  components positive is a 50-50 mixture of both positive and negative polarizations relative to an axis normal to the initial polarization direction with the  $x$  component negative.

Thus if we know that the initial spin is polarized as above, and we now measure the polarization in the direction of the classically expected final polarization, we will indeed find that the final spin is 100% polarized as expected from the classical analysis if the particle went through the path that interacts with the spin. But if the particle went through the other path and did not affect the spin at all, the spin is completely unpolarized with respect to this new axis, 50% positive and 50% negative.

This is thus only a “partial which path” experiment with partial dephasing.. The initial and final states of the spin before and after the rotation are very different and distinguishable classically. But quantum-mechanically they are not orthogonal. The overlap defines a domain where it is impossible to determine “which path” and interference will still be observed.

#### IV. DETAILED QUANTUM MECHANICS OF NEUTRINO DETECTOR

We now apply the general which-path formalism developed above to a neutrino - detector system. The wave function for the initial state of neutrino and detector can be written

$$\Psi_i(\nu, D) = \sum_{k=1}^{N_\nu} |\nu(E_\nu, m_k, \vec{P}_k), D_i(E_i)\rangle \quad (4.1)$$

where  $N_\nu$  is the number of neutrino mass states,  $E_\nu$ ,  $m_k$  and  $\vec{P}_k$  denote the neutrino energy, mass and momentum and  $D_i(E_i)$  is the initial state of the detector with energy  $E_i$ . If the detector is a muon detector the final detector state after neutrino absorption is

$$\Psi_f(\mu^\pm, D) = \sum_{k=1}^{N_\nu} |\mu^\pm(E_\mu, \vec{P}_\mu), D_{kf}^\mp(E - E_\mu)\rangle \quad (4.2)$$

where  $E_\mu$  and  $\vec{P}_\mu$  denote the muon energy and momentum,  $D_{kf}^\mp$  is the final detector state produced in the “path  $k$ ”; i.e. after the absorption of a neutrino with mass  $m_k$  and emission of a  $\mu^\pm$ , and  $E = E_\nu + E_i$  is the total energy which is conserved in the transition.

The transition in the detector occurs on a nucleon, whose co-ordinate is denoted by  $\vec{X}$ , and involves a charge exchange denoted by the isospin operator  $I_\mp$  and a momentum transfer  $\vec{P}_k - \vec{P}_\mu$ . The detector transition matrix element is therefore given by

$$\langle D_{kf}^\mp | T^\mp | D_i \rangle = \langle D_{kf}^\mp | I_\mp e^{i(\vec{P}_k - \vec{P}_\mu) \cdot \vec{X}} | D_i \rangle \quad (4.3)$$

The overlap between the final detector wave functions after the transitions absorbing neutrinos with masses  $m_k$  and  $m_j$  is then

$$\langle D_{kf}^\mp | D_{jf}^\mp \rangle = \langle D_i | e^{i(\vec{P}_j - \vec{P}_k) \cdot \vec{X}} | D_i \rangle \quad (4.4)$$

If the quantum fluctuations in the position of the active nucleon in the initial state of the detector are small in comparison with the oscillation wave length,  $\hbar/(\vec{P}_j - \vec{P}_k)$ ,

$$|\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i | |\vec{X}|^2 | D_i \rangle \ll 1 \quad (4.5)$$

$$\langle D_{kf}^\mp | D_{jf}^\mp \rangle \approx 1 - (1/2) \cdot |\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i | |\vec{X}|^2 | D_i \rangle \approx 1 \quad (4.6)$$

There is thus effectively a full overlap between the final detector states after absorption of different mass neutrinos, and a full coherence between the neutrino states with the same energy and different momenta.

The total energies of the final muon and detector produced after absorption of neutrinos with different energies are different. These muon-detector states are thus orthogonal to one another and there is no coherence between detector states produced by the absorption of neutrins with different energies.

## V. CONCLUSIONS

### A. What we know from simple quantum mechanics

Neutrinos propagate from the source to the detector as ordinary Dirac particles moving freely in space if they are not interacting with matter. They do not get lost in transit and the relative number of the different mass eigenstates is the same at the detector as at the source. Only the relative phase between the different mass eigenstates can change in the propagation from the source to the detector .

The observation of a difference between upward and downward going atmospheric neutrinos measured in the same detector can have only two possible explanations.

1. At least two different neutrinos with different masses are emitted from the source and observed in the detector, and the detector is sensitive to the relative phases of the waves arriving from neutrinos with different masses. These relative phases increase monotonically with distance as a well-known function of the unknown neutrino mass differences, thereby producing oscillations in the signal observed at the detector as a function of distance. The experimental results place constraints on the values of the neutrino mass differences and the couplings of the different neutrino mass states to the source and the detector (mixing angles in the language of the standard model).

2. The neutrinos traveling through the earth do not propagate freely, but interact with

matter. This is generally known as the MSW effect. These interactions can change the relative magnitudes as well as the relative phases of the neutrino mass eigenstates reaching the detector. They can transfer momentum, but they conserve energy like a ball bouncing elastically against the earth.

All these conclusions depend only upon quantum mechanics.

### **B. What we think we know from the standard model**

In the standard model all the neutrinos observed so far in experiments originate in a source from weak decays or  $W$  and  $Z$  exchanges and are detected via  $W$  or  $Z$  exchange in a detector. The couplings of the three neutrino mass eigenstates to the three charged leptons and the  $W$  is described by a  $3 \times 3$  unitary matrix analogous to the CKM matrix in the quark sector. These are usually described in terms of mixing angles.

### **C. What we don't know and need to determine from experiment**

The masses of the three types of neutrinos and the mixing angles describing their couplings to the  $W$  are completely unknown and are free parameters in the standard model. We really do not know if the standard model relations between couplings are really valid and whether new physics beyond the standard model might influence these relations. However, we emphasize that there is no justification for believing that new physics beyond the standard model can violate quantum mechanics. Thus the conclusions from quantum mechanics described above hold even if the standard model is not valid.

### **D. Energy-Momentum Kinematics**

We now use the above considerations to specify the relevant scales in neutrino oscillation experiments. Consider a neutrino emitted from a macroscopic source whose size is described by a linear dimension  $S$ , and detected by a macroscopic detector at a distance  $D \gg S$  from

the source. The knowledge of the source position leads to uncertainties in the initial source momentum, the momentum transfer and the neutrino momentum.

$$\delta p_\nu \approx \frac{\hbar}{S} \quad (5.1)$$

The energy of the source before the emission of the neutrino can be measured in principle with arbitrary precision. The energy after the emission can be measured during the time of flight of the neutrino from source to detector. This leads to an uncertainty in energy transfer and the neutrino energy

$$\delta E_\nu \approx \frac{\hbar c}{D} \ll c \delta p_\nu \quad (5.2)$$

The uncertainty in the square of the neutrino mass is then given by

$$\delta(m_\nu)^2 \cdot c^4 = 2p_\nu \cdot \delta p_\nu \cdot c^2 + 2E_\nu \cdot \delta E_\nu \approx 2p_\nu \cdot c^2 \cdot \left( \frac{\hbar}{S} + \frac{\hbar}{D} \right) \approx 2p_\nu \cdot c^2 \cdot \frac{\hbar}{S} \quad (5.3)$$

Interference effects can be observed at the detector between the contributions from neutrino states with different masses if the squared mass difference is less than this value (5.3). The uncertainty in neutrino mass arises from the uncertainty in the neutrino momentum. Eq. (5.2) shows that the uncertainty in the neutrino energy is negligible. Thus any coherence observed at the detector between amplitudes from neutrinos with different masses must come from states with the same energy and different momenta.

The relative phase between two neutrino waves with the same energy, masses  $m_1$  and  $m_2$  and momenta  $p_1$  and  $p_2$  changes in traversing a distance  $D$  by the amount

$$\delta\phi(D) = (p_1 - p_2) \cdot \frac{D}{\hbar} = \frac{(p_1^2 - p_2^2)}{(p_1 + p_2)} \cdot \frac{D}{\hbar} \approx \frac{(m_2^2 - m_1^2) \cdot c^4}{2p_\nu} \cdot \frac{D}{\hbar} \quad (5.4)$$

For this phase to be of order unity and give rise to observed neutrino oscillations,

$$m_2^2 - m_1^2 \approx \frac{2\hbar p_\nu}{Dc^4} \ll \frac{2\hbar p_\nu}{Sc^4} \approx \delta(m_\nu)^2 \quad (5.5)$$

This squared-mass difference (5.5) is much less than the lower limit on detectable mass difference imposed by the uncertainty condition (5.3). The momentum difference between

the different mass eigenstates having the same energy is much smaller than the momentum uncertainty produced by the localization of the source. Thus the neutrino mass difference needed to produce oscillations with wave lengths of the order of the source-detector distance  $D$  cannot be detected in any experiment in which the distance from the source to the detector is much larger than the size of the source.

The wave length of the neutrino oscillations is given directly by eq. (5.4).

For an example showing characteristic numbers, the neutrino momentum from a pion decay at rest is  $\approx 30$  MeV/c or  $3 \times 10^7$  ev/c. If there are two neutrino masses of 1 and 2 ev. their momentum difference if they have the same energy is

$$p_1 - p_2 \approx \frac{(m_2^2 - m_1^2)c^2}{p} = 10^{-7} \text{ ev/c} \quad (5.6)$$

Since  $\hbar c \approx 2 \times 10^{-7}$  ev  $\times$  meters, the oscillation wave length will be of order 2 meters and knowing the source position with a precision of more than two meters will prevent the measurement of this momentum difference. If the two neutrino masses are 0.1 and 0.2 ev. these numbers scale by a factor of 100 and  $p_1 - p_2 \approx 10^{-9}$  ev/c and the oscillation wave length is 200 meters.

This effectively says it all for neutrino propagation in free space between a source and detector whose size and distance satisfy the condition that the distance between source and detector is much greater than the size of the source. The point source approximation is good. Except for the case of matter-induced oscillations and the MSW effect or for the case of propagation through external fields there is no need to engage in more complicated descriptions to obtain the desired results.

## VI. ACKNOWLEDGMENTS

It is a pleasure to thank Maury Goodman, Yuval Grossman, Boris Kayser, Lev Okun and Leo Stodolsky for helpful discussions and comments.

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